Appendix D

Equations of incompressible resistive MHD

Both $\mathbf{v}$ and $\mathbf{B}$ are solenoidal

$$\nabla\cdot \mathbf{v} = \nabla\cdot \mathbf{B} = 0 \tag{D.1}$$

The definition of fluid vorticity closely parallels that of current density via Ampere's Law

$$\omega = \nabla\times \mathbf{v}, \quad \mathbf{j} = \nabla\times \mathbf{B} \tag{D.2}$$

The vorticity evolves in time according to a generalized Navier-Stokes equation that includes the effect of the Lorentz force term (j x B)

$$\partial_t\omega - \nabla\times(\mathbf{v} \times \omega + \mathbf{j} \times \mathbf{B}) = \mu_\nu(-1)^{\nu-1}\Delta^\nu\omega \tag{D.3}$$

Where μ_ν is the fluid viscosity and the exponent ν determines the type of diffusion operator that is used. $\nu = 1$ gives usual Navier-Stokes diffusion. $\nu = 2$ gives “hyperdiffusion”

The magnetic field also evolves in time according to a convection-diffusion equation:

$$\partial_t \mathbf{B} - \nabla\times(\mathbf{v} \times \mathbf{B}) = \eta_\nu (-1)^{\nu-1} \Delta^\nu \mathbf{B} \tag{D.4}$$

Where η_ν is the electrical resistivity of the fluid. Be aware that equations $D.3$ and $D.4$ are nonlinearly coupled through the convection terms.

Appendix E

Vector field visualization using a complex color code

Appendix F

Direction of future research